

4.1

Solve Simple Equations

Pep rallies are a great way to build school spirit by cheering on your favourite school teams and clubs. Suppose your student council raises \$500 in a school spirit fundraising drive to buy school T-shirts to give away at a pep rally and puts you in charge of purchasing. How can your understanding of equations help you determine how many shirts you can buy?



Investigate

How can you use a simple equation to solve a problem?

1. Byron spent a total of \$11 on two magazines. The cost of one magazine is \$5. You can use an **equation** to find the cost of the other magazine.
 - a) Choose a variable to represent the unknown.
 - b) Write an equation to represent this situation.
 - c) What value of the variable makes the equation true? Describe the math operation(s) you used to find the value.
2. Kelly spent a total of \$10 on a pen and two mechanical pencils. The pen cost \$4 and the pencils each cost the same amount.
 - a) Choose a variable to represent the unknown.
 - b) Write an equation to represent this situation.
 - c) What value of the variable makes the equation true? Describe the math operation(s) you used to find the value.
3. **Reflect**
 - a) How can you use mathematical operations to solve equations?
 - b) Explain how you can verify your answers.

equation

- a mathematical statement that says two expressions are equal
- $3x + 3 = 2x - 1$ is an equation

Example 1 Solve Equations Involving Adding and Subtracting

Solve each equation.

- a) $x + 4 = 13$
- b) $x - 8 = 2$
- c) $-4 + x = -1$

To solve an equation means to find the value of the variable that makes the statement true.

Solution

a) Method 1: Inspection

Sometimes you can solve an equation by inspection, which means just looking at it and applying number sense.

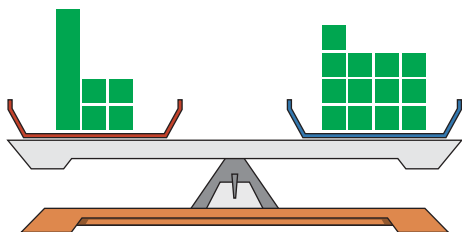
$$\begin{aligned}x + 4 &= 13 \\x &= 9\end{aligned}$$

A number plus 4 gives 13.
The number must be 9.

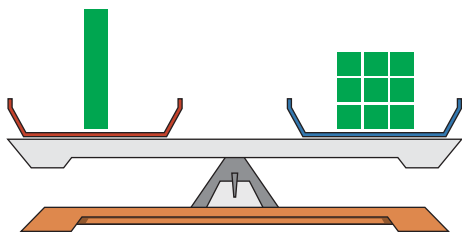
The **solution** is $x = 9$.

Method 2: Balance Method

Think of an equation as being like a balance, with the equal sign representing the centre. You can use algebra tiles to represent the quantity on each side.



To find what x equals, remove 4 unit tiles. You must do this to both sides to keep the equation balanced.



The solution is $x = 9$.

Method 3: Use the Opposite Operation

An equation is still true if you apply identical operations to both sides.

$$\begin{aligned}x + 4 &= 13 \\x + 4 - 4 &= 13 - 4 \\x &= 9\end{aligned}$$

If I subtract 4 on the left, I'll be left with just x , because $+4 - 4$ makes zero. That means I'll need to subtract 4 on the right, too.

The solution is $x = 9$.

solution

- the value of the variable that makes an equation true



Go to
www.mcgrawhill.ca/links/principles9 and follow the links to find a Web site that provides an interactive model of the balance method for solving equations.

Literacy Connections

Follow these guidelines to communicate your solution effectively:

- Show all steps clearly.
- Use one equal sign per line.

b) Method 1: Inspection

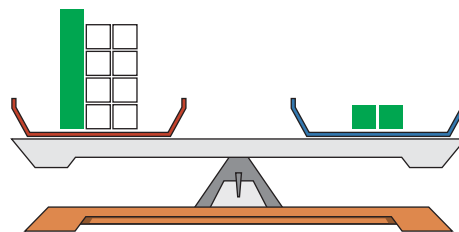
$$\begin{aligned}x - 8 &= 2 \\x &= 10\end{aligned}$$

A number minus 8 gives 2.
The number must be 10.

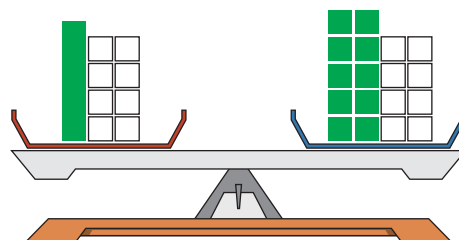
The solution is $x = 10$.

Method 2: Balance Method

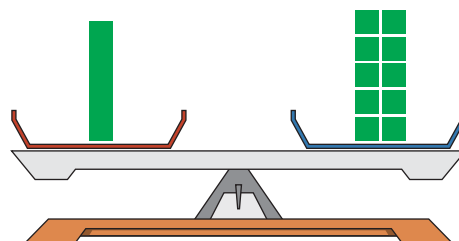
There are eight negative unit tiles on the left that need to be removed. You cannot do this immediately because there are none on the right side.



Add eight zero pairs to the right side first.



Now remove eight negative unit tiles from each side to solve for the unknown.



The solution is $x = 10$.

Method 3: Use the Opposite Operation

$$\begin{aligned}x - 8 &= 2 \\x - 8 + 8 &= 2 + 8 \\x &= 10\end{aligned}$$

I need to add 8 to both sides.

The solution is $x = 10$.

- c) Solving equations by inspection is efficient if the numbers are easy to work with. Otherwise, it is better to apply opposite operations. When applying opposite operations, you are isolating the variable.

$$\begin{aligned}-4 + x &= -1 \\-4 + 4 + x &= -1 + 4 \\x &= 3\end{aligned}$$

I need to add 4 to both sides to get x by itself.

The solution is $x = 3$.

Literacy Connections

To isolate the variable means to express an equation so that the variable appears alone on one side of the equation. For example,

$$\begin{aligned}-4 + x &= -1 \\-4 + 4 + x &= -1 + 4 \\x &= 3\end{aligned}$$

The variable x has been isolated on the left side of the equation.

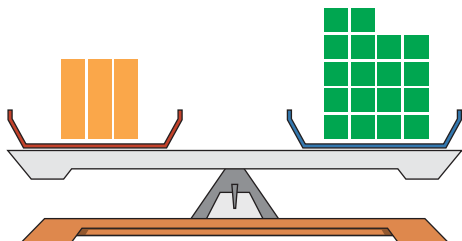
Example 2 Solve Equations Involving Multiplying and Dividing

Solve each equation.

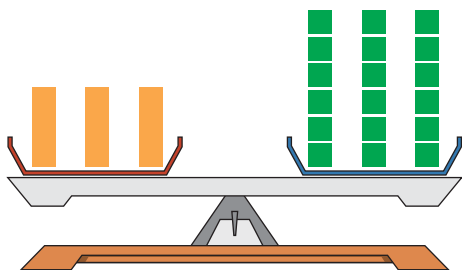
a) $3y = 18$ b) $\frac{n}{3} = -4$ c) $-v = 9$

Solution

a) Method 1: Balance Method



Divide both sides into three equal groups.



The solution is $y = 6$.

Method 2: Use the Opposite Operation

$$3y = 18$$

$$\frac{3y}{3} = \frac{18}{3}$$

$$y = 6$$

3 times a number gives 18. I can divide both sides of the equation by 3 to find the unknown.

b) Use the Opposite Operation

$$\frac{n}{3} = -4$$

$$3 \times \frac{n}{3} = 3 \times (-4)$$

$$n = -12$$

n divided by 3 gives -4 . To find the number, I need to undo the division. The opposite of dividing is multiplying. I need to multiply both sides by 3.

c) Use the Opposite Operation

$$-v = 9$$

$$\frac{-v}{-1} = \frac{9}{-1}$$

$$v = -9$$

$-v$ means the same as $-1v$. To undo multiplication by -1 , I need to divide by -1 .

root (of an equation)

- the value of the variable that makes an equation true
- the same as the solution of an equation

Literacy Connections

Finding the root of an equation means the same thing as solving an equation.

For example:

$$9 - x = 2$$

The root of this equation is 7 because it makes the statement true: $9 - 7 = 2$.

Example 3 Solve a Two-Step Equation

Find the **root** of the equation and check the solution.

$$5x + 25 = 500$$

Solution

Method 1: Use Opposite Operations, Pencil and Paper

Isolate the variable term first.

$$5x + 25 = 500$$

$$5x + 25 - 25 = 500 - 25 \quad \text{Subtract 25 from both sides.}$$

$$5x = 475$$

$$\frac{5x}{5} = \frac{475}{5}$$

Divide both sides by 5.

$$x = 95$$

The root of the equation is $x = 95$.

Check if this answer is correct by substituting $x = 95$. Evaluate the left side (L.S.) and the right side (R.S.) of the equation. Both sides of the equation must have the same value.

$$\begin{aligned} \text{L.S.} &= 5x + 25 & \text{R.S.} &= 500 \\ &= 5(95) + 25 \\ &= 475 + 25 \\ &= 500 \\ &\text{L.S.} = \text{R.S.} \end{aligned}$$

The root is 95.

When $x = 95$, L.S. = 500 and R.S. = 500.

To check a solution, I must make sure that the left side and the right side have the same value.

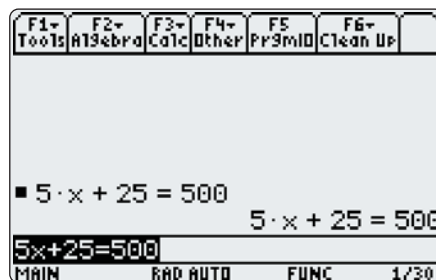
Therefore, $x = 95$ is correct.

Method 2: Use Opposite Operations, Computer Algebra System (TI-89)

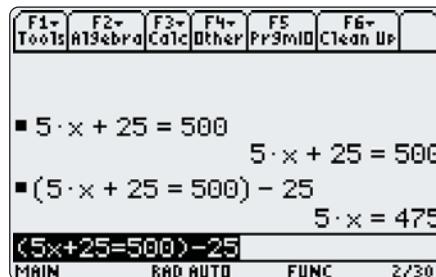
In the Home screen, type the equation $5x + 25 = 500$.

Then, press **(ENTER)**.

Notice that the equation has been repeated in the command line.



Isolate the variable term first. Press the left cursor key. Insert a bracket before the 5. Cursor right, and place another bracket after the 500. Then, subtract 25. Press **(ENTER)**.



I am subtracting, so I use the subtract key, not the negative key.

Copy and paste the new equation onto the command line:

- Use the up cursor key to highlight the new equation.
- Press \blacklozenge \uparrow for [COPY].
- Use the down cursor key to return to the command line.
- Press \blacklozenge ESC for [PASTE].

Instead of copying and pasting, I could retype the equation in the command line.

Put brackets around the equation and divide by 5. The brackets are needed for a Computer Algebra System (CAS) to understand where the equation begins and ends.

- Press ENTER .

The solution is $x = 95$.

Check if this solution is correct by substituting $x = 95$ into the equation. The CAS result will be either true or false. Press the following:

$(5 \times 95 + 25 = 500)$
 $(5 \times 95 + 25 = 500)$ ENTER

Technology Tip

When you use a CAS to solve an equation step by step, the equation should look simpler after each step. If it does not, check that you have performed the correct step. You may need to backtrack and correct your error. A CAS makes this easy, using the COPY and PASTE commands.

The *true* result verifies that $x = 95$ is a correct solution.

Example 4 Modelling With Equations

A student council has raised \$500 in a school spirit fundraising drive. The council decides to use the funds to buy school T-shirts to give away at a pep rally. If the T-shirts cost \$6 each, how many can the student council buy?

Solution

Use n to represent the number of T-shirts the student council can buy.

$$\begin{aligned} 6n &= 500 \\ \frac{6n}{6} &= \frac{500}{6} && \text{Divide both sides by 6.} \\ n &= 83.\bar{3} \end{aligned}$$

\$6 times the number of T-shirts, n , must give \$500. I can solve this equation for n .

Since you cannot buy part of a T-shirt, round to 83. The student council can buy 83 T-shirts.

Key Concepts

- To solve an equation means to find the value of the variable that makes the statement true. This is also called finding the root of the equation.
- To solve a one-step equation, isolate the variable by performing the opposite operation.
- In a two-step equation, there is more than one term on one side, so isolate the variable term first by adding or subtracting. Then, divide by the coefficient of the variable term.

For example:

$$2x - 7 = 9$$

$$2x - 7 + 7 = 9 + 7 \quad \text{Add or subtract to isolate the variable term. BEDMAS}$$

$$2x = 16$$

$$\frac{2x}{2} = \frac{16}{2} \quad \text{Multiply or divide to solve for the variable. BEDMAS}$$

$$x = 8$$

- Check a solution to an equation by substituting the root into the left side and the right side of the equation. Both sides must be equal.

For the example above:

Substitute $x = 8$.

$$\begin{aligned} \text{L.S.} &= 2(8) - 7 & \text{R.S.} &= 9 \\ &= 16 - 7 \\ &= 9 \end{aligned}$$

$$\text{L.S.} = \text{R.S.}$$

This is like applying BEDMAS in reverse.

Communicate Your Understanding

- C1** Describe the first step you would take to solve each equation.

a) $k - 5 = -11$

b) $3m = 18$

c) $\frac{x}{5} = -4$

d) $5n + 75 = 225$

- C2** Which is the correct solution to $2x + 5 = 21$? Explain how you can tell without solving the equation.

A $x = 7$

B $x = 8$

C $x = 9$

- C3** A high school football team has raised \$1000 to spend on team jackets. The cost is \$50 per jacket. Which equation can be used to solve for the number of jackets the team can buy? Explain.

A $50 = 1000n$

B $50n = 1000$

C $1000n = 50n$

Practise

For help with questions 1 to 3, see Example 1.

1. Solve by inspection.

a) $x - 5 = 4$

b) $m + 8 = 11$

c) $y - 3 = 0$

d) $h + 2 = 6$

2. Solve using the balance method.

a) $x + 5 = 12$

b) $x - 6 = 7$

c) $y + 3 = 10$

d) $y - 4 = 2$

3. Solve using opposite operations.

a) $x + 7 = 12$

b) $n - 8 = 11$

c) $-5 + y = -2$

d) $-9 + h = -6$

For help with questions 4 and 5, see Example 2.

4. Solve using the balance method.

a) $3x = 12$

b) $5y = 20$

c) $\frac{n}{3} = 8$

d) $-2k = 16$

5. Solve using opposite operations.

a) $4z = -24$

b) $\frac{h}{-5} = -6$

c) $-6c = -42$

d) $-9u = 45$

For help with questions 6 and 7, see Example 3.

6. Find the root of each equation using paper and pencil. Apply opposite operations. Check each root.

a) $7x - 4 = 10$

b) $7k + 2 = 16$

c) $-p + 7 = 0$

d) $-12g - 33 = 0$

7. Use a CAS to solve. Apply opposite operations. Check each solution.

a) $k - 4 = -9$

b) $6x = -30$

c) $\frac{q}{7} = 2$

d) $2y - 7 = 9$

e) $-3w - 1 = 14$

f) $2q - 9 = -13$

8. Solve using the method of your choice. Check your answers.

a) $p + 9 = -2$

b) $-5x = 35$

c) $\frac{u}{4} = -8$

d) $6r + 3 = 33$

e) $10c - 6 = -16$

f) $-3v + 6 = -9$

For help with questions 9 and 10, see Example 4.

9. At a bake sale, pies cost \$7 each. One customer buys \$84 worth of pies.

a) Write an equation to model the number of pies the customer bought.

b) Solve the equation.

10. A hockey team has \$700 to buy new jerseys. Ice-wear, a jersey supplier, charges \$50 per jersey. How many new jerseys can the team buy?
- Write an equation that models the number of jerseys the team can afford.
 - Solve the equation. Write a conclusion to the problem.

Connect and Apply

11. Copy the following solution. Write a short explanation beside each step. The first step has been done for you.

<i>Step</i>	<i>Explanation</i>
$3x - 8 = 7$	
$3x - 8 + 8 = 7 + 8$	Add 8 to both sides.
$3x = 15$	
$\frac{3x}{3} = \frac{15}{3}$	
$x = 5$	

12. Solve each equation. Express fraction answers in lowest terms. Check each solution.

- $2k - 7 = -8$
- $3x + 8 = 2$
- $4m - 6 = 12$
- $-9u + 8 = 23$

13. Solve each equation. Express fraction answers in lowest terms.

- $8r - \frac{3}{2} = -15$
- $-10h - 6 = -\frac{2}{5}$

14. **Chapter Problem** Your first task as producer of Canadian Superstar is to rent a theatre for the first event, a singing competition. Rental includes lunch and snacks for the competitors. Details for the two best choices are shown:

Royal James Hall
\$50 per person

Broadway Nights
\$1000 plus \$30 per person

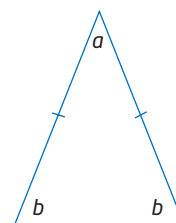
You have \$2000 in your budget for this event. You would like to begin the competition with as many contestants as you can afford.

- Write an equation to model the cost for renting Royal James Hall. Solve the equation.
- The cost for renting Broadway Nights can be modelled by the equation $C = 1000 + 30n$. Explain why this equation correctly gives the cost, C , in dollars, for n contestants.
- Use the total amount budgeted ($C = 2000$) to solve this equation for n .
- Which hall should you rent? Explain.

15. A hockey team has \$700 to buy new jerseys. In question 10, you found how many jerseys the team could buy from Ice-wear. Another jersey supplier, Rink Rat, sells jerseys for \$40 each plus a \$75 logo design fee.
- Write an equation that models the number of jerseys the team can afford.
 - Solve the equation.
 - From which supplier should the team buy their jerseys? Explain.
 - What other factors might influence this decision?
16. Marcel has \$40 to spend on amusement park rides. Tickets cost \$1.50 without a special membership pass, or \$1.25 with a membership pass. A membership pass costs \$5.00. Should Marcel buy a membership pass? Use mathematical reasoning to justify your answer.

Achievement Check

17. In an isosceles triangle, the equation $a + 2b = 180^\circ$ relates the two equal angles and the third angle.
- Use this equation to find the value of a when
 - $b = 25^\circ$
 - $b = 100^\circ$
 - Use this equation to find the value of b when
 - $a = 40^\circ$
 - $a = 100^\circ$
 - What is the maximum possible whole-number value of a ? Justify your answer.
 - What is the maximum possible whole-number value of b ? Justify your answer.



Extend

18. Justine's mother is building an ultralight airplane. The fuel tank is made of plastic and has a mass of 5000 g. Each litre of gasoline has a mass of 840 g. The total mass of the fuel plus the tank cannot exceed 21 800 g.
- Write an equation that models the number of litres of gasoline that the tank may hold.
 - Solve the equation to determine the number of litres of fuel in a full tank.
19. Refer to question 14.
- Would you change your decision if your budget were doubled for this event? Explain.
 - Is there a budget value for which it does not matter which hall you rent? If so, find the value.



20. **Math Contest** If m and n are positive integers and $m + n = 6$, which is a possible value for $3m - 2n$?

A 4

B -4

C 0

D 2

E -2